

Linking Scales in Computations: from Microstructure to Macro-scale Properties

## On the influence of damage evolution in an incompressible material with matrix displaying tension-compression asymmetry

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### Abstract

A strong difference between the behavior in tension versus compression is observed at the polycrystal level, if either twinning or non-Schmid type slip are contributors to plastic deformation at the single crystal level. Despite recent progress in modeling the effects of this asymmetry in yielding, its influence on damage evolution remains a challenge. In a recent paper [1] we presented a new constitutive model for voided polycrystal that incorporates the effects of the tension-compression asymmetry of the incompressible matrix on the overall dilatational plastic behavior. In this contribution, this model is used to investigate the influence of the tension-compression asymmetry of the matrix on void evolution for uniaxial loading conditions. It is shown that if the matrix tensile strength is higher than its compressive strength, void growth and damage distribution are similar to that in classical materials obeying Gurson's [2] criterion. On the other hand, for certain porous polycrystals in which the matrix tensile strength is lower than its compressive strength, void growth rate is much slower. Damage distribution is significantly different; the location of the zone of maximum porosity shifts from the center of the specimen outwards. Furthermore, the influence of the evolving microstructure on void growth is studied. It is shown that void growth is significantly affected by the rate of change of the matrix strength differential with plastic strain.

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## 1. Introduction

Significant progress has been made in understanding and modeling the micro-mechanics of ductile fracture in porous polycrystalline materials. Most of the available theories of dilatational plasticity and viscoplasticity make use of the assumption that the matrix (void-free material) is incompressible and displays the same response in tension and compression. This is the case for the classical Gurson [2] model and its various extensions (e.g. [3-7]). However, even in the absence of voids hexagonal close packed metals (hcp) are incompressible but display tension-compression asymmetry (see [8-10] for examples of asymmetry in hcp metals). Thus, the yield criterion for the matrix should be a criterion that represents the plastic response of the virgin polycrystal. Such an approach was very recently adopted by Cazacu and Stewart [1], who extended Gurson's [2] analysis of a hollow sphere to the case when the matrix has different yield strengths in tension versus compression and obeys the isotropic form of the Cazacu et al. [11] yield criterion. The resulting plastic potential for the porous polycrystal depends on all three invariants of stress. In this contribution we briefly present a coupled elastic-plastic damage model with yielding described by [1] and new findings concerning the influence of the matrix tension-compression asymmetry on void evolution and the location of the zone corresponding to maximum damage in round tensile specimens subject to uniaxial tension. Two types of specimens and boundary conditions are considered. One is a classical notched tensile specimen for which necking is induced by a small initial imperfection; the second is a round bar tensile specimen in which striction is initiated by the imposed boundary conditions. Our calculations show that for certain porous polycrystals having the matrix tensile strength lower than its compressive strength, void growth rate is much slower than in a material obeying Gurson-Tvergaard-Needleman (GTN) criterion [3]. Furthermore, the location of the maximum void volume fraction is shifted from the center of the specimen outwards. In hcp polycrystals such as Ti, Mg, Zr, the interplay between deformation twinning and slip leads to evolution of the matrix tension-compression strength ratio with plastic deformation. To investigate this relevant aspect of microstructure evolution, simulations for a material characterized by a matrix tension-compression ratio that depends on the equivalent plastic strain are also carried out. It is shown that void evolution and damage distribution depend in a significant manner on the rate of change of the matrix strength asymmetry.

## 2. Constitutive model

Using the kinematic non-linear homogenization approach of Hill-Mandel (see [12-13]), Cazacu and Stewart [1] developed an analytic yield criterion for a porous material containing randomly distributed spherical voids in an isotropic, incompressible matrix that displays tension-compression asymmetry and obeys the isotropic form of the Cazacu et al. [11] yield criterion. The yield criterion [1] is of the form:

$$m^2 \sum_{i=1}^3 \left( \frac{|\sigma'_i| - k\sigma'_i}{\sigma_T} \right)^2 + 2f \cosh \left( -z_s \frac{3P}{2\sigma_T} \right) - (1 + f^2) = 0 \quad (1)$$

where  $f$  is the void volume fraction;  $\sigma'_1, \sigma'_2, \sigma'_3$  are the principal values of the deviator of the Cauchy stress tensor,  $\sigma' = \sigma + P\mathbf{I}$ ; and  $P = -(\sigma : \mathbf{I})/3$ , with  $\mathbf{I}$  being the second-order identity tensor. In Eq. (1),  $k$  is a strength-differential material parameter, which is expressible only in terms of the ratio between the matrix tensile strength,  $\sigma_T$ , and the matrix compressive strength,  $\sigma_c$ ; i.e.

$$k = (1 - h)/(1 + h) \text{ with } h = \sqrt{2 - (\sigma_T / \sigma_c)^2} / \sqrt{2(\sigma_T / \sigma_c)^2 - 1} \quad (2)$$

From Eq. (2), it follows that for the material parameter  $k$  to be real,  $1/\sqrt{2} \leq \sigma_T/\sigma_C \leq \sqrt{2}$ . The material constant  $m$  in Eq. (1) is defined as

$$m = 3/\sqrt{2(3k^2 - 2k + 3)}. \quad (3)$$

The parameter  $z_s$  involved in the expression of the criterion (1) depends on the sign of the pressure. Its expression is

$$z_s = \begin{cases} 1 & \text{if } P \geq 0 \\ \sigma_T/\sigma_C & \text{if } P < 0 \end{cases}. \quad (4)$$

If the matrix material has no tension-compression asymmetry in its plastic responses ( $\sigma_T = \sigma_C$ ), then the coefficient  $z_s = 1$  and  $m = \sqrt{3}/2$ , so that the criterion (1) reduces to Gurson's criterion [2] automatically. If the void volume fraction,  $f$ , is equal to zero, the yield criterion [1] reduces to the matrix yield criterion (i.e. to the isotropic form of the Cazacu et al. [11] yield criterion).

Fig. 1 shows a comparison between finite-element (FE) cell calculations and the projection in the deviatoric plane of the analytical yield loci given by Eq. (1) for porous isotropic materials with matrix having either yield strengths in tension less than in compression ( $\sigma_T/\sigma_C = 0.82$  or  $k = -0.3098$ ) or yield strengths in tension greater than in compression ( $\sigma_T/\sigma_C = 1.21$  and  $k = 0.3098$ ) or no tension-compression asymmetry ( $\sigma_T/\sigma_C = 1$ ;  $k = 0$ , GTN material). In all the unit cell calculations, a spherical void ( $f = 0.01$ ) is embedded in an elastic-plastic cylindrical matrix with elastic constants  $E/\sigma_T = 800$  and  $\nu = 0.32$  obeying the isotropic form of the Cazacu et al. [11] criterion. The applied loading is axisymmetric i.e.  $\sigma = \sigma_{11}(e_1 \otimes e_1 + e_2 \otimes e_2) + \sigma_{33}(e_3 \otimes e_3)$ , where  $(e_1, e_2, e_3)$  form an orthonormal basis. Such loading, corresponds to either a positive (for  $\sigma_{33} > \sigma_{11}$ ) or negative ( $\sigma_{33} < \sigma_{11}$ ) value of the third invariant of the stress deviator  $J_3 = \sigma'_1 \sigma'_2 \sigma'_3$ ; the von Mises equivalent stress is:  $\sigma_e = |\sigma_{11} - \sigma_{33}|$ , and the pressure  $P = -(\sigma_{33} + 2\sigma_{11})/3$ . The effects of the tension-compression asymmetry of the matrix are assessed by conducting simulations for both tensile and compressive loadings. Because Cazacu and Stewart's criterion [1] depends on all principal values of the stress deviator  $\sigma'$ , the yield loci for the materials characterized by tension-compression asymmetry ( $\sigma_T/\sigma_C \neq 1$  or  $k \neq 0$ ) are no longer invariant with respect to the transformation  $(P, \sigma') \rightarrow (P, -\sigma')$  as it is the case with the GTN material ( $\sigma_T/\sigma_C = 1$  or  $k = 0$ ; see Fig.1). Furthermore, the yield loci are no longer symmetric with respect to the deviatoric axis ( $P = 0$ ). Indeed, according to the criterion (1), yielding occurs when  $P = 2\sigma_C \ln(f)/3$  for tensile hydrostatic loading while, for compressive hydrostatic loading, yielding occurs at  $P = -2\sigma_T \ln(f)/3$  (exact solutions). In Cazacu and Stewart [1], it was shown that the model predictions can be further improved if the expression (1) is modified to include additional parameters,  $q_i$ ,  $i = 1 \dots 3$  (in the same spirit as Tvergaard, [14]) as follows:

$$\varphi = m^2 \sum_{i=1}^3 \left( \frac{|\sigma'_i| - k\sigma'_i}{\bar{\sigma}} \right)^2 + 2q_1 f \cosh \left( -q_2 z_s \frac{3P}{2\bar{\sigma}} \right) - (1 + q_3 f^2). \quad (5)$$

and  $\bar{\sigma}$  denotes the tensile effective stress for the matrix material. Matrix hardening is considered to be governed by the effective plastic strain,  $\bar{\varepsilon}^p$ .

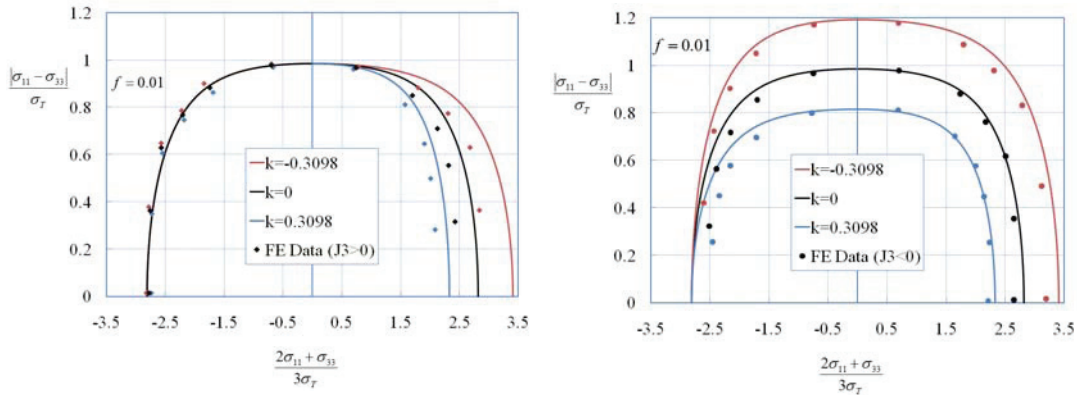


Fig. 1. FE yield points and theoretical yield surfaces according to criterion (Eq.(1)) for three porous materials ( $f = 0.01$ ) with incompressible matrix having either the yield strength in tension less than the yield strength in compression displaying tension-compression asymmetry ( $\sigma_T/\sigma_C = 0.82$ ,  $k = -0.3098$ ), or yield strength in tension greater than the yield strength in compression ( $\sigma_T/\sigma_C = 1.21$ ;  $k = 0.3098$ ) or same yield in tension and compression ( $k=0$ ;  $\sigma_T/\sigma_C = 1$ ; GTN material) for axisymmetric loading with a)  $\sigma_{33} > \sigma_{11}$  ( $J_3 \geq 0$ ), b)  $\sigma_{33} \leq \sigma_{11}$  ( $J_3 \leq 0$ ).

The following power law is considered

$$\bar{\sigma} = \alpha (\varepsilon_0 + \bar{\varepsilon}^p)^n, \quad (6)$$

with  $n$  being a constant hardening exponent, while the parameters  $\alpha$ , and  $\varepsilon_0$  with dimension of stress and strain, respectively characterize the tensile yield value. The evolution of the void volume fraction is obtained using mass conservation

$$\dot{f} = (1-f) \mathbf{D}^p : \mathbf{I}, \quad (7)$$

where  $\mathbf{D}^p$  is the plastic part of the macroscopic strain. The rate of the microscopic effective plastic strain  $\dot{\bar{\varepsilon}}^p$  is obtained, assuming the equivalence of microscopic and macroscopic inelastic work and associated flow rule, as

$$\bar{\sigma} \dot{\bar{\varepsilon}}^p (1-f) = \boldsymbol{\sigma} : \mathbf{D}^p, \quad (8)$$

$$\dot{\bar{\varepsilon}}^p = \frac{\boldsymbol{\sigma} : \mathbf{D}^p}{(1-f)\bar{\sigma}} = \dot{\lambda} \frac{\boldsymbol{\sigma} : \frac{\partial \phi}{\partial \boldsymbol{\sigma}}}{(1-f)\bar{\sigma}}, \quad (9)$$

where  $\dot{\lambda}$  is the plastic multiplier rate.

The main objective of this work is to assess the influence of the tension-compression asymmetry of the matrix on void growth. The calculations are terminated when a critical void volume fraction is reached marking the beginning of void coalescence that would further lead to the initiation of a macroscopic crack. In all the calculations, the only parameter that is varied is the strength-differential parameter  $k$  (see Eq. (2)). The material parameters that are kept the same are: the elastic properties of the matrix (Young modulus  $E = 300$  MPa; Poisson coefficient  $\nu = 0.3$ ), the matrix tensile yield strength ( $\alpha = 1.8$  MPa;  $\varepsilon_0 = 0.0027$ ;  $n = 0.1$ ), the initial void volume fraction  $f_0 = 0.04$ , and Tvergaard's parameters,  $q_1 = 1.5$ ,  $q_2 = 1$ ,  $q_3 = q_1^2$ . Since void nucleation is not considered and all the materials studied have identical tensile yield strengths, any differences in damage evolution are due solely to the differences between the matrix tension-compression asymmetry of the respective porous materials. As already mentioned, if the matrix does not display tension-compression asymmetry ( $\sigma_T/\sigma_C = 1$  so  $k = 0$ ), then the yield criterion (5) reduces to the GTN model. The numerical values for the matrix elastic properties, matrix yield stress in tension and matrix hardening in tension are the same as those given by Tvergaard and Needleman [3] and Aravas [15]. This allows for partial verification of the FE implementation of the Cazacu and Stewart [1] model through qualitative comparison with Tvergaard and Needleman [3] results.

### 3. Analysis of void evolution in a notched tensile bar

First, we analyze void evolution in a long, notched specimen with circular cross-section. An initial geometric imperfection is used to induce necking in the specimen (see Fig.2). The specimen has an initial length of  $2L_0$  and initial radius of  $R_0$  with  $L_0/R_0 = 4$ , with  $R_0$  set to unity. A cylindrical reference coordinate system (radial coordinate  $R$ , axial coordinate  $Z$ ) is used for the analysis. The kinematic boundary conditions are: symmetry about  $R = 0$  and symmetry about  $Z=0$ ; all the nodes along  $Z = L_0$  are pulled in the  $Z$  direction while the nodes in the radial direction are free. The FE mesh for the eighth of the bar analyzed is shown in Fig. 2b. It consists of 5472 hexahedric elements with reduced integration (ABAQUS C3D8R). The simulations are carried out using a User Material Subroutine (UMAT) that we developed for the coupled elastic-plastic damage model described in Section 2 and implemented in the FE code ABAQUS [16]. A fully implicit integration algorithm was used for solving the governing equations.

#### 3.1. Evolution of the void volume fraction for materials characterized by a fixed tension/compression strength ( $\sigma_T/\sigma_C = \text{constant}$ )

Round tensile bars made of three isotropic materials containing randomly oriented spherical voids for which the matrix has the yield in tension larger than the yield in compression with  $\sigma_T/\sigma_C = 1.238$ , and  $\sigma_T/\sigma_C = 1.41$ , respectively and  $\sigma_T/\sigma_C = 1$  (GTN material-no strength differential effects) are examined. For these materials, the corresponding values of the strength differential (SD) parameter are:  $k = 0.355$ ,  $k = 0.9$ , and  $k = 0$  (GTN material-no strength differential effects). Displacement boundary conditions are applied at the end of the specimens until  $f = 15.5\%$ . Fig. 3 shows contours of constant void volume fraction,  $f$ . For all materials, the maximum void volume fraction develops at the center of the specimen, but for the material with matrix characterized by  $k = 0.9$  ( $\sigma_T/\sigma_C = 1.41$ ), the extent of the zone of maximum damage is smaller compared with either the material with  $k = 0.355$  ( $\sigma_T/\sigma_C = 1.238$ ) or the GTN material ( $k = 0$ ). Note that for the material with the highest contrast between matrix strengths ( $k = 0.9$ ) the damage isosurfaces become square with rounded corners whereas for lower contrast the isosurfaces are oval. Yet, as long as the matrix yield in tension is greater or equal than that in compression ( $\sigma_T \geq \sigma_C$ ), the void distribution in the specimen is qualitatively similar irrespective of the value of the tension-compression ratio  $k$ .

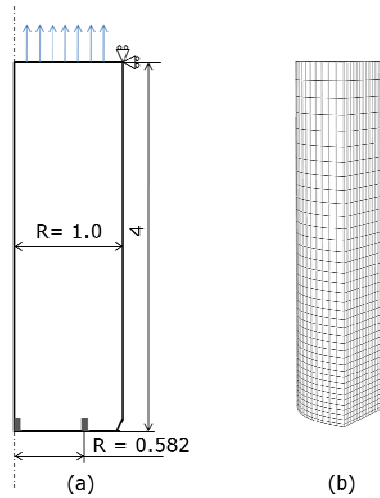


Fig. 2. (a) Geometry and boundary conditions for the tensile test with a notched specimen of circular cross section; (b) FE mesh of the eighth of the specimen model.

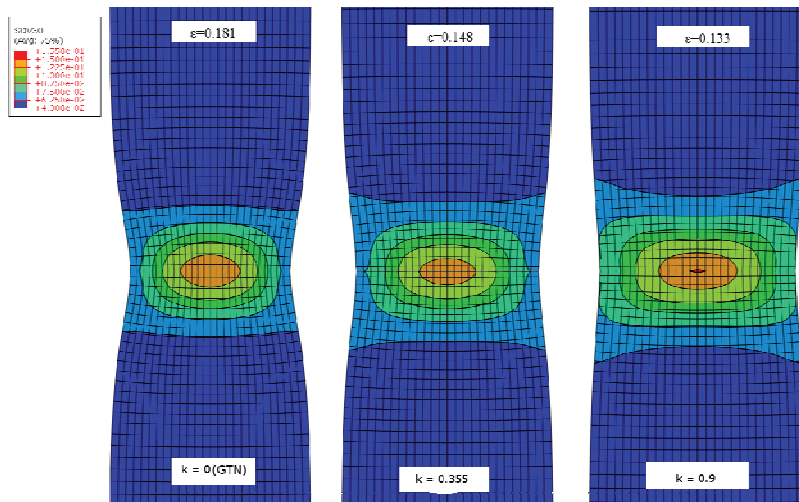


Fig. 3. Contours of constant void volume fraction,  $f$ , for materials with matrix having the yield in tension larger than the yield in compression  $k = 0.355$  ( $\sigma_T/\sigma_C = 1.238$ ) and  $k = 0.9$  ( $\sigma_T/\sigma_C = 1.41$ ), in comparison with a GTN material ( $k = 0$ ).

Simulations have also been performed for different values of  $k$  negative, i.e. for materials with matrix yield in tension less than in compression ( $\sigma_T < \sigma_C$ ). The contours correspond to the end of each test. The results shown in Fig.4 demonstrate an important influence of the tension/compression asymmetry on the void volume fraction distribution. For  $k = -0.355$  ( $\sigma_T/\sigma_C = 0.81$ ), the void volume fraction map is qualitatively similar to that for  $k \geq 0$ , with the maximum void volume fraction occurring at the center of the specimen. For  $k = -0.544$  ( $\sigma_T/\sigma_C = 0.75$ ), there is a much larger spread of the zone of maximum

damage than in the case when  $k = -0.355$  ( $\sigma_T/\sigma_C = 0.81$ ); as  $k$  further decreases ( $k = -0.686$  corresponding to  $\sigma_T/\sigma_C = 0.72$ ), the maximal damage zone extends to the entire section. For  $k = -0.9$  ( $\sigma_T/\sigma_C = 0.71$ ), the maximal damage zone location shifts from the center of the specimen outwards. A clear correlation between the value of the tension-compression asymmetry ratio and the location of the maximal damage zone is observed (Fig.4). It is worth pointing that  $k = -0.544$  corresponds to a tension-compression asymmetry ratio  $\sigma_T/\sigma_C$  of 0.75 while  $k = -0.9$  corresponds to  $\sigma_T/\sigma_C = 0.71$ . Thus, a difference of less than 5% in the matrix tension/compression ratio leads to a very strong difference in void distribution.

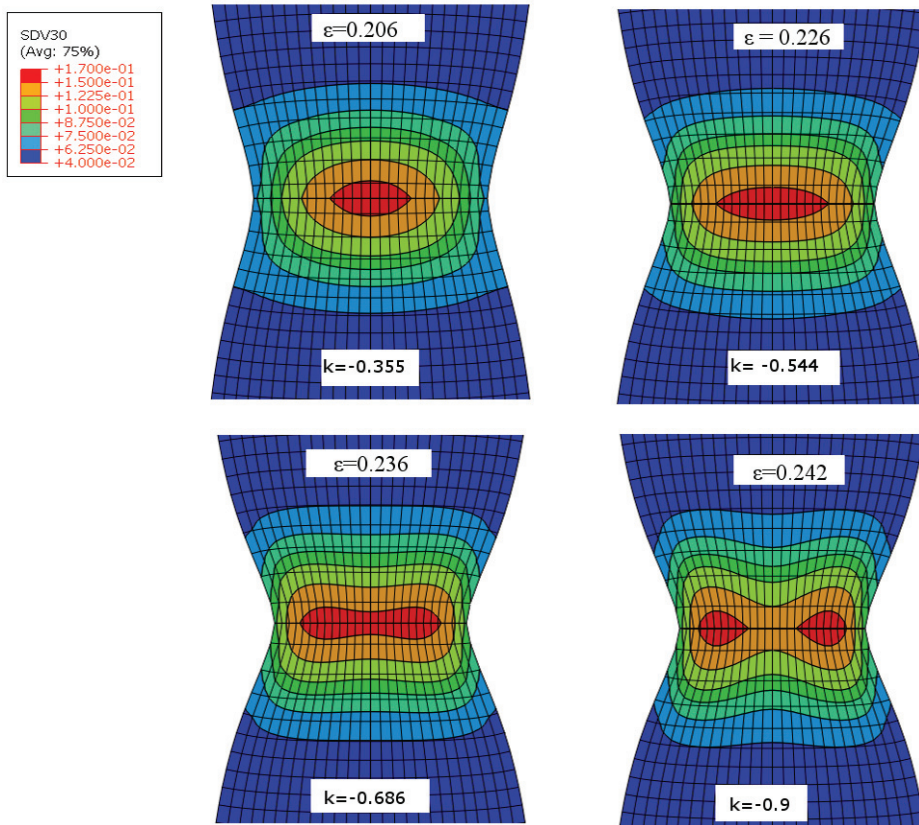


Fig. 4. Contours of constant void volume fraction,  $f$ , for materials with matrix having the yield in tension less than the yield in compression:  $k = -0.355$  ( $\sigma_T/\sigma_C = 0.81$ ),  $k = -0.544$  ( $\sigma_T/\sigma_C = 0.75$ ),  $k = -0.686$  ( $\sigma_T/\sigma_C = 0.72$ ) and  $k = -0.9$  ( $\sigma_T/\sigma_C = 0.71$ ), corresponding to the end of the test (i.e. for  $f_{max} = 17\%$ ).

To better quantify the observed shift, in Fig. 5 is shown the normalized void volume fraction ( $f/f_{\max}$ ) as a function of the distance from the center of the specimen ( $R/R_{\max}$ ) for materials with different tension/compression asymmetry ratios. In each case, the void distribution corresponds to the end of the test (all simulations are terminated at  $f_{\max} = 15\%$ ;  $R_{\max}$  denotes the radius of the specimen at the end of a given test). Dimensionless quantities for damage and radial distance have been used in order to allow comparison between the void fraction distributions corresponding to different materials. The trends illustrated in Fig. 3 and 4 are confirmed. It is very interesting to note that the void volume fraction distribution is not simply dictated by the sign of  $k$  (i.e. whether the matrix tensile strength in tension is greater than in compression), but also depends on the absolute value of this parameter. Indeed, the material characterized by  $k = -0.355$  ( $\sigma_T/\sigma_C = 0.81$ ) has a response much more similar to the material for which  $k > 0$  ( $\sigma_T > \sigma_C$ ) than that of the material for which  $k < -0.686$  ( $\sigma_T < \sigma_C$ ).

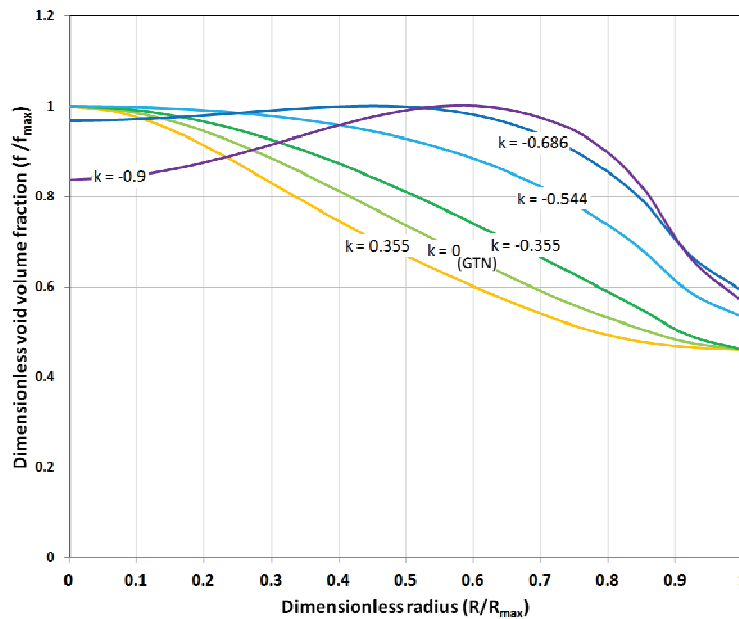


Fig. 5. Void volume fraction as a function of the distance from the centre of the specimen at the end of the test for materials with matrix with different tensile/compression asymmetry ratios:  $k = 0.355$  ( $\sigma_T/\sigma_C = 1.24$ ),  $k = 0$  (GTN material),  $k = -0.355$  ( $\sigma_T/\sigma_C = 0.81$ ),  $k = -0.544$  ( $\sigma_T/\sigma_C = 0.75$ ),  $k = -0.686$  ( $\sigma_T/\sigma_C = 0.72$ ) and  $k = -0.9$  ( $\sigma_T/\sigma_C = 0.71$ ). Maximum void volume fraction is shifted from the center if  $k < -0.544$ .

### 3.2. Evolution of the void volume fraction for materials characterized by evolving microstructure (tension/compression strength ratio $\sigma_T/\sigma_C$ variable)

In all the simulations presented so far, the strength differential parameter  $k$  was considered to be a constant. Yet, in some materials, the tension/compression asymmetry evolves with accumulated plastic strain. For example, experimental results on high-purity  $\alpha$ -titanium show that compression twinning is responsible for the tension-compression asymmetry of the matrix, which evolves as a result of the evolving microstructure (see the marked evolution in texture under uniaxial compression reported in Nixon et al., [8] which results in a variation of the SD ratio from zero to  $\sim -0.9$ ). Although, the

deformation behavior of HCP materials have received increased attention recently (e.g. studies on zirconium [9] or titanium and its alloys, e.g. [10] and [17], just to name a few very recent contributions) experimental data on damage and its evolution are not available.

To study the influence of the evolving microstructure on void growth, the following two evolution laws, with  $k$  ranging from zero ( $\sigma_T/\sigma_C = 1$ ) at the onset of plastic flow to a maximum value of  $k_{\max} = -0.9$  ( $\sigma_T/\sigma_C = 0.709$ ) will be considered:

$$k(\bar{\varepsilon}) = a + \frac{b}{1 - \exp(\bar{\varepsilon} - \bar{\varepsilon}_1)/\bar{\varepsilon}_2)} \quad (10)$$

and

$$k(\bar{\varepsilon}) = a + \frac{b}{1 - \exp(\bar{\varepsilon} - \bar{\varepsilon}_3)/\bar{\varepsilon}_4)} \quad (11)$$

□

with  $a = 0.90416$ ,  $b = 0.009321$ ;  $\bar{\varepsilon}_1 = 0.15$ ,  $\bar{\varepsilon}_2 = 0.02788$ , while  $\bar{\varepsilon}_3 = 0.4$ ,  $\bar{\varepsilon}_4 = 0.06788$ .

□

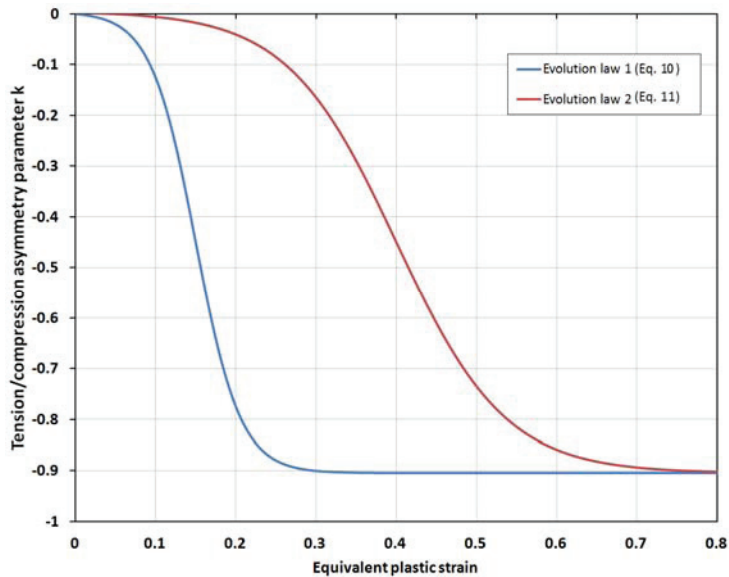


Fig.6. Evolution laws for  $k$  as a function of the equivalent von Mises macroscopic plastic strain  $\bar{\varepsilon}$ .

Note that the only difference between these two exponential laws is the rate of change of  $k$  (see Fig.6). Fig. 7a shows the contour of constant void volume fraction at the end of the test for a material with matrix characterized by a fixed value of the strength ratio,  $k = -0.9$  ( $\sigma_T/\sigma_C = 0.71$ ) while Figure 7b and 7c present the void isocontours for a material with evolving strength ratio  $k$  according to Eq. (10) and Eq. (12), respectively. For all materials, the calculations are terminated at a void volume fraction  $f_{\max} = 0.14$ . Although, the limiting (final value) of the tension-compression asymmetry ratio or parameter  $k$  for both materials with evolving microstructures is the same ( $k = -0.9$ ), there is a very striking difference in the

void distribution within the respective bars. For the material for which  $k$  evolves according to Eq. (10) i.e. there is a fast decrease of  $k$  from zero ( $\sigma_T/\sigma_C = 1$ ) to the final value  $k = -0.9$  ( $\sigma_T/\sigma_C = 0.71$ ), the zone of maximum damage is shifted from the center of the specimen outward, i.e. the response is quite similar to that of the material with  $k = -0.9$  (constant tension-compression strength ratio  $\sigma_T/\sigma_C = 0.71$ ). For the second evolution law, the decrease from  $k = 0$  to  $k = -0.9$  is slower than for the first evolution law. The void volume fraction distribution within the specimen is totally different, the maximum damage zone develops at the center of the specimen, i.e. similar to a GTN material ( $\sigma_T/\sigma_C = 1$ ) i.e. with  $k = 0$  (constant). It can be concluded that the evolution of the parameter  $k$  strongly influences the void volume fraction distribution. Fig. 8 show the local damage evolutions in an element at the center of the specimen and at mid-radius ( $R/R_0 = 0.582$ , see also Fig. 2a) for the same three materials. These results confirm the paramount influence of the rate of change of the parameter  $k$  with the von Mises equivalent plastic strain. Indeed, in a material for which  $k$  evolves according to Eq. (10), the void growth in both the center element ( $R=0$ ) and mid-radius element is quite similar to that obtained for a material with matrix characterized by the limiting value  $k = -0.9$  (constant tension-compression strength ratio  $\sigma_T/\sigma_C = 0.71$ ). Note the intersection between the void growth curves for the two elements, which explain the shift in the location of the zone of maximum damage of Fig. 7(b). It is worthwhile to note that for the material with matrix tension-compression asymmetry evolving according to Eq. (11), the void growth rate is similar to that corresponding to a material with no tension-compression asymmetry (i.e.  $\sigma_T/\sigma_C = 1$  so  $k = 0$ ) up to an equivalent plastic strain of about 4%, while beyond this strain level the void growth rate becomes slower and comparable to that of a material with matrix characterized by a constant tension-compression ratio  $\sigma_T/\sigma_C = 0.71$  ( $k = -0.9$ ). *In conclusion, the results presented in Fig 7-8 demonstrate that the final void volume fraction distribution could not be predicted based on either the initial or final value of  $k$ , the rate of change of the tension-compression ratio being of foremost importance.*

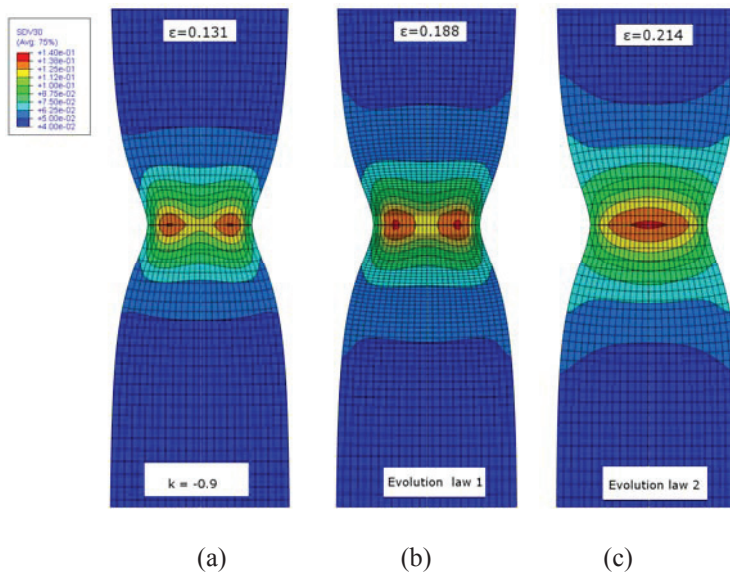


Fig.7. Contours of constant void volume fraction,  $f_v$ , for materials with matrix having (a) constant strength ratio,  $k = -0.9$  ( $\sigma_T/\sigma_C = 0.71$ ); (b) evolving microstructure with  $k$  following the evolution law (10), and (c) evolving microstructure with  $k$  following evolution law (11) corresponding to the end of the test (i.e. for  $f_{max} = 15\%$ ).

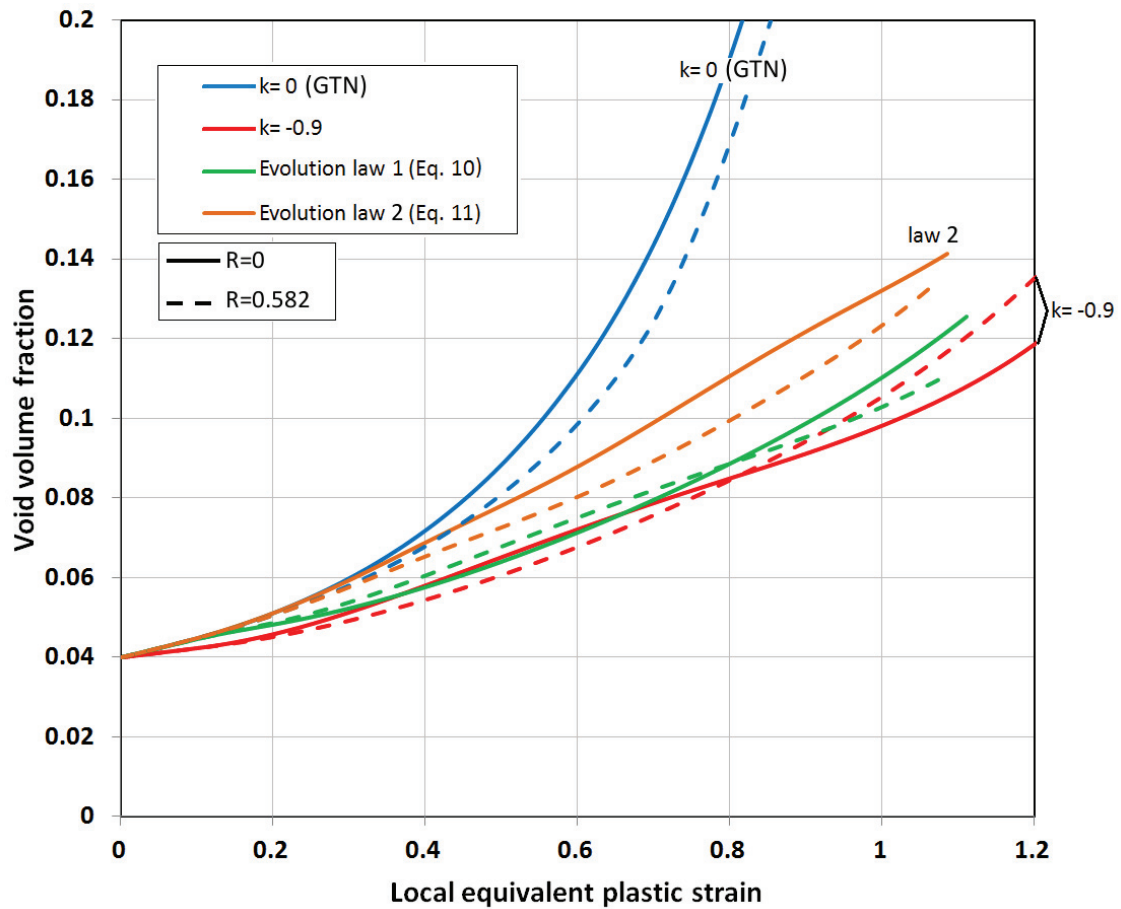


Fig.8. Evolution of the void volume as a function of the local equivalent plastic strain at the center of the specimen ( $R = 0$ ) and at  $R/R_0 = 0.582$ , for materials with matrix having constant strength ratio with  $\sigma_T/\sigma_C = 0.71$  ( $k = -0.9$ ) and GTN material ( $\sigma_T/\sigma_C = 1$  so  $k = 0$ ), respectively in comparison with the void growth in materials with evolving microstructure characterized by a strength differential ratio  $k$  following the evolution law (10) and (11), respectively.

The simulations of the notched round tensile bars presented have shown the great influence of the matrix tension/compression asymmetry on void growth and overall damage patterns. In these simulations, the damage is triggered by the notch, which may in turn introduce perturbations in the numerical calculations. To demonstrate that the new features associated to damage growth result from the matrix tension-compression asymmetry and are not due to the position or type of the notch, in the following section simulations of tensile tests on unnotched round bars will be carried out. The non-homogeneity in deformation will be triggered by the applied boundary conditions.

#### 4. Analysis of void evolution in a notched tensile bar

We analyze void evolution in a round tensile specimen of the same dimensions and aspect ratio as considered in the Section 3. However, there is no more geometric imperfection (notch) in the specimen, and the boundary conditions have been changed to initiate striction. The geometry and boundary conditions are described in Fig.9. Axial displacement is imposed, while radial displacement is set to zero. The FE mesh for the quarter of the bar analyzed consists of 7298 elements with reduced integration (C3D8R). Simulations are conducted using the same UMAT for the coupled elastic-plastic damage model presented in Section 2.

##### 4.1. Evolution of the void volume fraction for materials characterized by a fixed tension/compression strength ( $\sigma_T/\sigma_C = \text{constant}$ )

Since there is no notch, damage takes longer to initiate but the finite element results have increased accuracy owing to the fact that no perturbations may influence the calculations. First, void growth in round tensile bars made of isotropic materials for which the matrix has the yield in tension larger than the yield in compression with  $k = 0.355$  ( $\sigma_T/\sigma_C = 1.24$ ),  $k = 0.544$  ( $\sigma_T/\sigma_C = 1.33$ ), and  $k = 0.9$  ( $\sigma_T/\sigma_C = 1.41$ ), respectively and for a GTN material with  $k = 0$  (no strength differential effects) is analyzed. For each material, calculations are terminated at  $f=15\%$ .

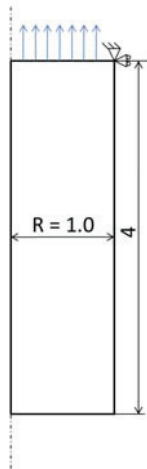


Fig. 9. Geometry and boundary conditions for the round bar tensile test specimen.

Fig. 10 shows contours of constant void volume fraction,  $f$ . As in the case of notched specimens made of the same materials, maximum damage occurs at the center of the specimen (see Fig. 3). Yet, without the notch, the results are smoother. Note that with increasing the contrast between the tensile and compressive strengths of the matrix (i.e. by increasing  $k$ ), the zone of maximum void volume fraction becomes smaller. However, the void distribution in the specimen is qualitatively similar irrespective of the value of the tension-compression ratio  $k$ .

Simulations have also been performed for different values of  $k$  negative, i.e. for materials with matrix yield in tension less than in compression ( $\sigma_T < \sigma_C$ ). The contours correspond to the end of each test (computations are terminated when  $f = 0.17$ ). The results shown in Fig.11-12 demonstrate the important influence of the value of the tension/compression asymmetry ratio on the void volume fraction distribution.

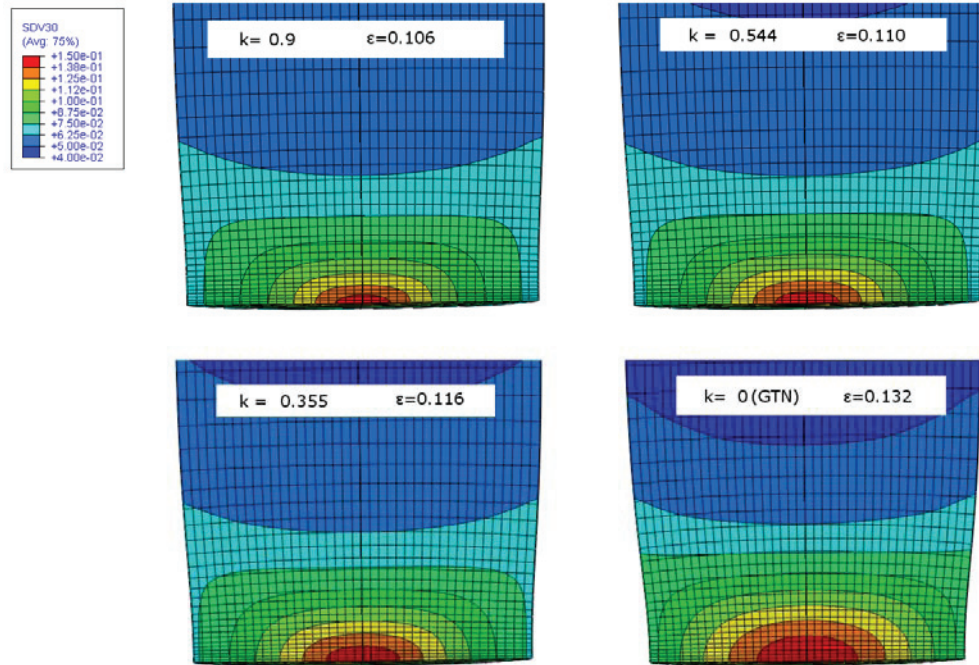


Fig.10. Contours of constant void volume fraction for materials with matrix characterized by different values of  $k$  positive:  $k = 0.9$  ( $\sigma_T/\sigma_C = 1.41$ ),  $k = 0.544$  ( $\sigma_T/\sigma_C = 1.33$ ),  $k = 0.355$  ( $\sigma_T/\sigma_C = 1.24$ ), in comparison with a GTN material ( $k = 0$ ). The maximum void volume fraction is the same for all simulations ( $f = 15\%$ ).

Without the notch, the results are smoother, but show the same trends observed for the notched specimens (see Fig.4-5). Note that for  $k = -0.686$  ( $\sigma_T/\sigma_C = 0.72$ ), the location of the zone of maximum damage shifts from the center of the specimen outwards, while the extent of this zone is reduced. This phenomenon takes place for  $k < -0.544$ , yet it is difficult however to determine precisely the value of the tension/compression asymmetry ratio at which this shift in the location of maximum damage is observed.

Results pointing to this shift (i.e. qualitatively analogous to those of Figures 5 and 12), have been obtained for different values of the strain hardening exponent and initial porosities values. Changes in those parameters have an influence only on the axial strain level attained in the tensile test when this event takes place, and on the value of the maximum void volume fraction.

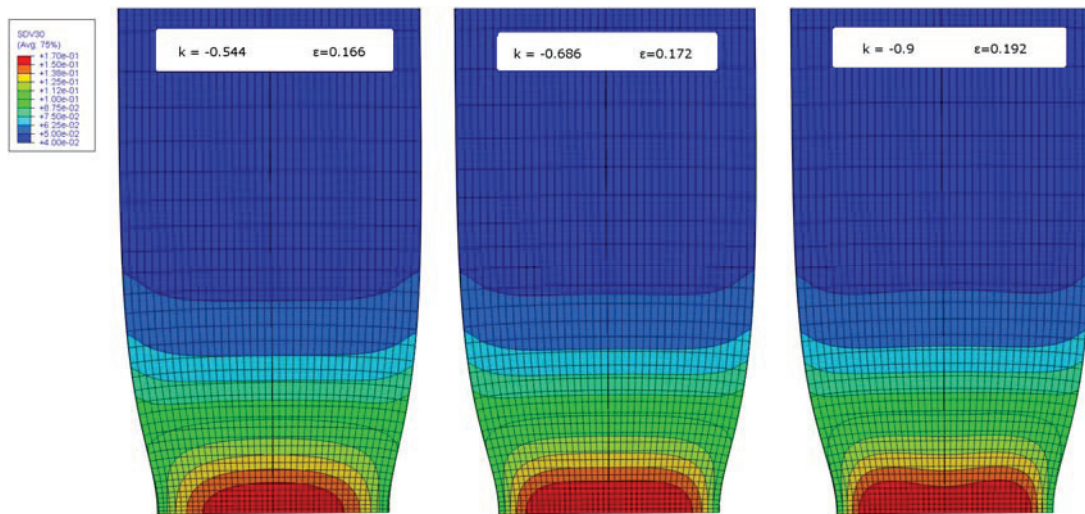


Fig. 11. Contours of constant void volume fraction for materials with matrix characterized by different values of  $k$  negative:  $k = -0.544$  ( $\sigma_T/\sigma_C = 0.75$ );  $k = -0.686$  ( $\sigma_T/\sigma_C = 0.72$ );  $k = -0.9$  ( $\sigma_T/\sigma_C = 0.71$ ).

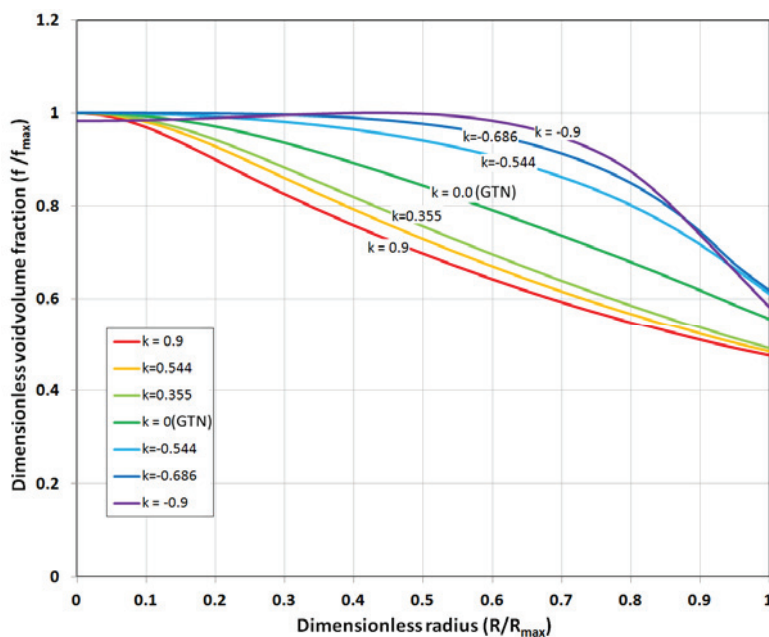


Fig.12. Void volume fraction as a function of the distance from the center of the specimen at the end of the test for materials with matrix characterized by different tensile/compression asymmetry ratios:  $k = 0.9$  ( $\sigma_T/\sigma_C = 1.41$ ),  $k = 0.544$  ( $\sigma_T/\sigma_C = 1.33$ ),  $k = 0.355$  ( $\sigma_T/\sigma_C = 1.24$ ),  $k = 0$  (GTN material),  $k = -0.544$  ( $\sigma_T/\sigma_C = 0.75$ );  $k = -0.686$  ( $\sigma_T/\sigma_C = 0.72$ );  $k = -0.9$  ( $\sigma_T/\sigma_C = 0.71$ ). Maximum void volume fraction is shifted from the center if  $k < -0.544$ .

#### 4.2. Evolution of the void volume fraction for materials characterized by evolving microstructure): application to titanium

As already mentioned, experimental data on damage and its evolution in hcp material are not available. Yet, to illustrate the unusual features of the damage growth to be expected in such materials, simulations are presented for a high-purity  $\alpha$  titanium material (for more details see [18]). Although, this material is anisotropic, the anisotropy in flow stresses is not pronounced (see [8]) so in the analysis that will be presented in the following, the material's anisotropy is neglected. The pre-necking uniaxial tensile and compressive stress-strain response corresponding to the rolling direction of the material (see Fig.13) is used to identify the law of evolution of the tension/compression asymmetry parameter,  $k$ , and the matrix tensile hardening law, respectively. Tvergaard's parameters have been set to  $q_1=1.5$ ,  $q_2=1.0$ ,  $q_3=q_1^2$ , while the initial void volume fraction was taken as  $f_0=0.04$ . The Young modulus and the Poisson coefficient were set to values typical for a titanium material:  $E=110000$  MPa and  $\nu=0.3$ .

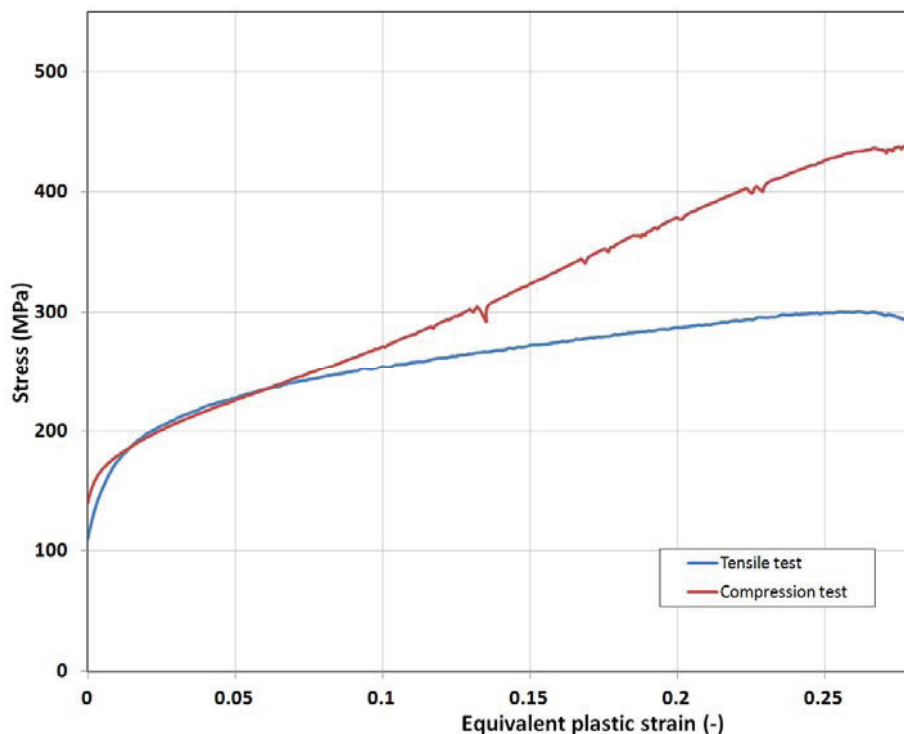


Fig. 13. Comparison between the tension and compression response in the rolling direction for a high-purity  $\alpha$ -titanium (after Nixon et al.[8]).

Although, initially there is no significant difference in yielding behavior, a very pronounced tension–compression asymmetry is observed after about 10% strain. Note the especially sharp difference in hardening evolution. While in tension, the material hardens gradually until plastic localization (diffuse necking) occurs at about 27% strain, in compression strain-hardening is strongly non-linear, with a very pronounced increase in hardening rate observed at about 10% strain. Additional tests, where the material

was deformed in compression up to 10%, 20%, 30%, and 40%, respectively were carried out and the post-test textures were measured. Significant texture evolution resulting from large grain rotations associated with twinning was observed in all samples. The microstructure evolution is captured in the model by the evolution of the parameter  $k$  (see Fig. 14). In Fig. 14, the  $k$ -values calculated at discrete levels of equivalent plastic strain using Eq. (2) are represented by symbols. This parameter evolves from 0 to -0.95; its evolution as a function of plastic strain was approximated by the following exponential law:

$$k(\bar{\varepsilon}) = c + \frac{d}{1 + \exp(\bar{\varepsilon} - \bar{\varepsilon}_n) / \bar{\varepsilon}_m} \quad (12)$$

with  $c = -0.995$ ,  $d = 1.003$ ,  $\bar{\varepsilon}_n = 0.1905$ , and  $\bar{\varepsilon}_m = 0.038$ .

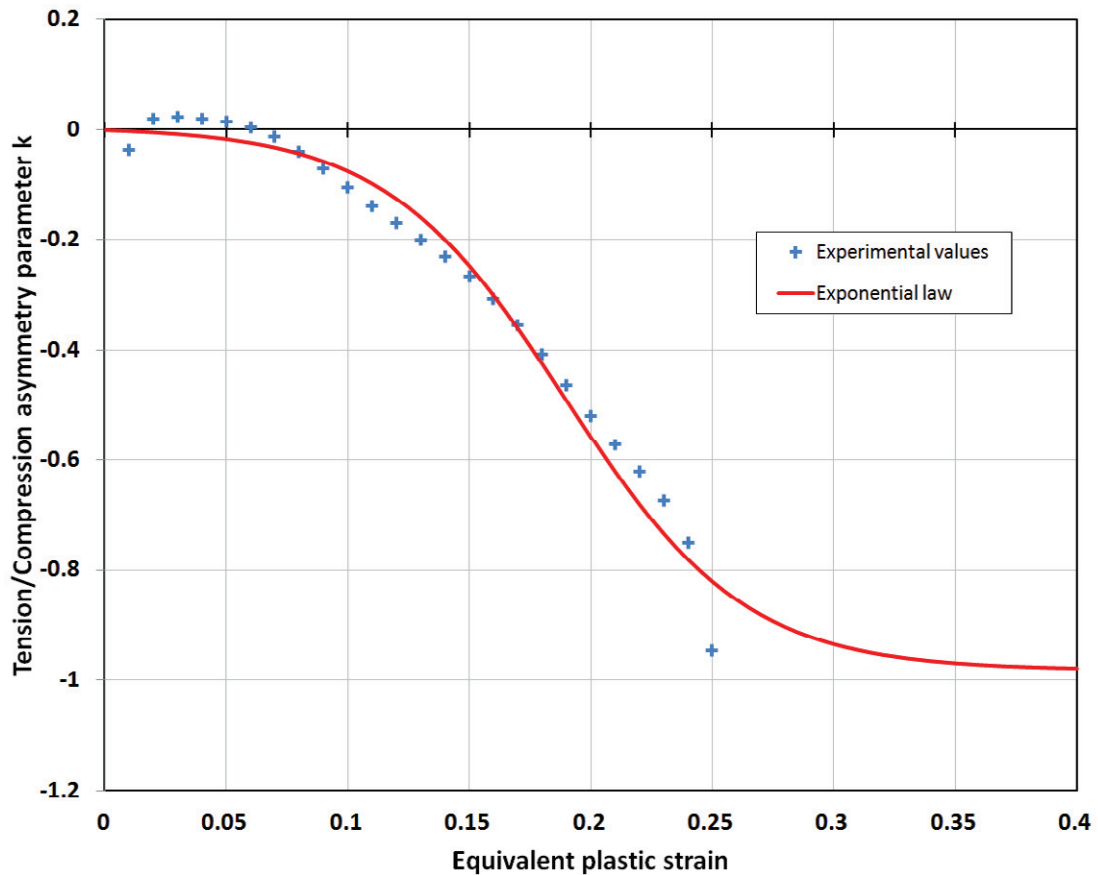


Fig. 14. Evolution law for  $k$  as a function of the equivalent plastic strain  $\bar{\varepsilon}$  for a high-purity  $\alpha$ -titanium (after Nixon et al., [8]).

As concerning the tensile hardening law for the matrix, the coefficients involved in the power law given by Eq. (6) are  $n = 0.17$ ,  $\alpha = 377.75 \text{ MPa}$ , and  $\varepsilon_0 = 0.000391$ . Fig.15 shows the void volume fraction distribution corresponding to different stages of the tensile test (several fixed values of the von Mises

effective strain  $\varepsilon$ ). Due to evolving microstructure, reflected in the evolution of the parameter  $k$ , at the end of the test ( $\varepsilon = 0.216$ ), the maximum void volume fraction is not at the center of the specimen.

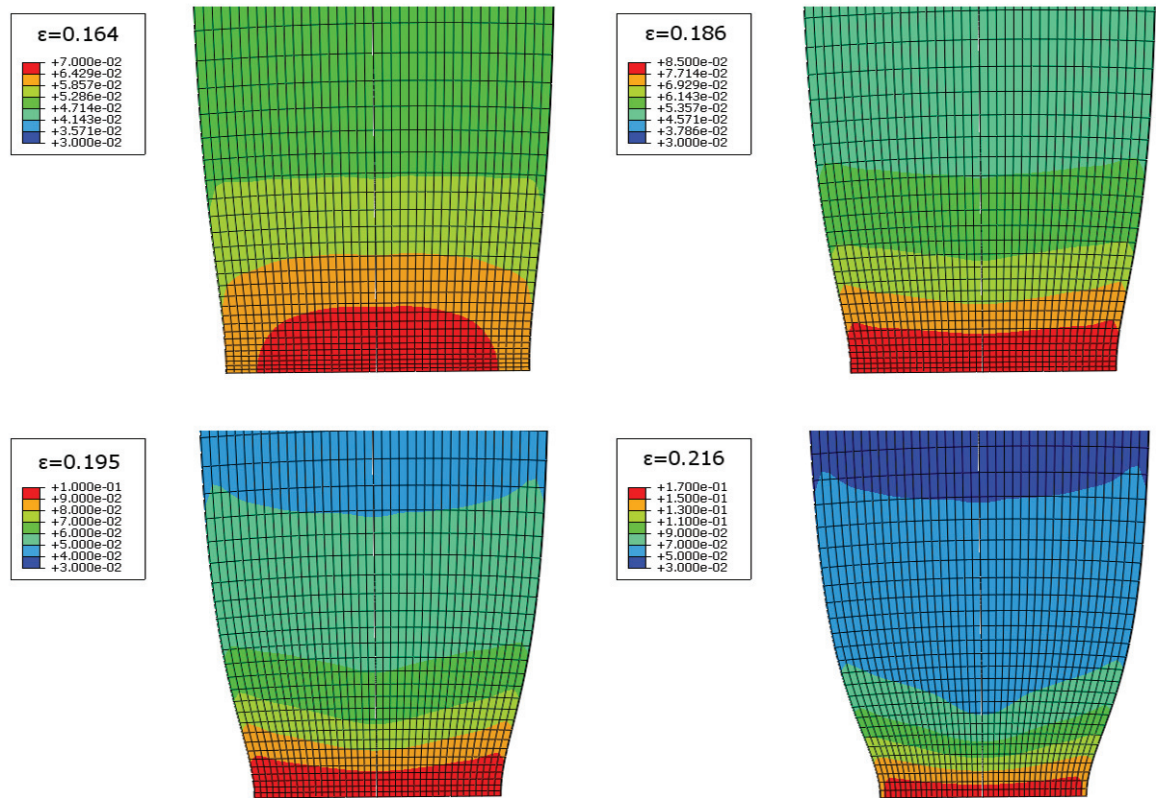


Fig. 15. Contours of constant void volume fraction corresponding to different stages of the test on a high-purity  $\alpha$ -titanium with matrix strength asymmetry characterized by the evolution law of Eq. (12).

## 5. Conclusions

In this contribution, using the recently developed model of Cazacu and Stewart [1], an analysis of the damage evolution in porous materials for uniaxial tensile loading conditions was presented. The simulations of the notched round tensile bars presented have shown the great influence of the matrix tension/compression asymmetry on void growth and overall damage distribution. In these simulations, the damage was triggered by the notch, which may in turn introduce perturbations in the numerical calculations. To demonstrate that the new features of damage growth that were observed result from the matrix tension-compression asymmetry and are not due to the position or type of the notch, simulations of tensile tests on unnotched round bars were also carried out.

All the numerical simulations (i.e. for both geometries considered) show that in porous metallic materials for which the tensile strength of the matrix is higher than its compressive strength, void growth and damage distribution are similar to that for materials that do not exhibit tension-compression asymmetry. On the other hand, for certain porous solids in which matrix tensile strength is lower than its compressive strength, novel and unexpected damage trends were revealed. Namely, damage is delayed and the void growth rate is much slower than in the classical cases presented so far in the literature, i.e. materials which do not display tension-compression asymmetry and are modeled using the GTN model. Furthermore, damage distribution is significantly different; the location of the zone of maximum porosity is shifted from the center of the specimen outwards. It is to be noted that even a slight tension-compression asymmetry ratio for the matrix (of about 0.7) leads to damage evolution that is strikingly different than that for a material with matrix that does not display strength differential effects. Results pointing to this shift have been obtained for different values of the matrix strain hardening exponent and initial porosities values. Changes in those parameters have an influence only on the axial strain level attained in the tensile test when this event takes place (see also [18]).

Furthermore, the influence of the evolving microstructure on void growth was studied. For this purpose, simulations were carried out by considering an evolving strength differential ratio for the matrix. It was shown that void growth and damage distribution is significantly affected by the rate of change of the strength differential ratio with plastic strain.

As already mentioned, although the deformation behavior of hcp materials have received increased attention, detailed experimental data of void growth that could allow validation of the numerical simulations presented are not available. Yet, to illustrate the unusual features of the damage growth to be expected in such materials, simulations were presented for a high-purity  $\alpha$ -titanium material, the strength ratio evolution considered corresponding to the rolling direction. The trends predicted may provide guidance for a systematic experimental investigation on damage by void growth in this material and others hcp materials as well.

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## References

- [1] Cazacu O, Stewart JB. Analytic plastic potential for porous aggregates with matrix exhibiting tension-compression asymmetry. *J Mech Phys Solids* 2009; **57**: 325–41.
- [2] Gurson AL. Continuum Theory of ductile rupture by void nucleation and growth – Part I. Yield criteria and flow rules for porous ductile media. *J Engng Mat Tech* 1977; **99**: 2–15.
- [3] Leblond J-B, Perrin G, Suquet P. Exact results and approximate models for porous viscoplastic solids. *Int J Plasticity* 1994; **10**: 213–25.
- [4] Tvergaard V, Needleman A. Analysis of the cup-cone fracture in a round tensile bar. *Acta Metall* 1984; **32**: 157–69.
- [5] Gologanu M, Leblond J-B, Perrin G, Devaux J. Recent extensions of Gurson's model for porous ductile metals. In : *Continuum Micromechanics*, Springer Verlag; 1994, p. 61-130.
- [6] Garajeu M, Suquet P. Effective properties of porous ideally plastic or viscoplastic materials containing rigid particles. *J Mech Phys Solids* 1997; **45**: 873-902.
- [7] Monchiet V, Cazacu O, Charkaluk E, Kondo D. Macroscopic yield criteria for plastic anisotropic materials containing spheroidal voids. *Int J Plasticity* 2008; **24**: 1158-89.
- [8] Nixon ME, Cazacu O, Lebensohn, RA. Anisotropic response of high-purity  $\alpha$ -titanium: Experimental characterization and constitutive modeling. *Int. J. Plasticity* 2010; **26**: 510-32.
- [9] Proust G., Tome, CN, Kaschner, GC. Modeling texture, twinning, and hardening evolution during deformation of hexagonal materials. *Acta Mater* 2007; **55**: 2137–48.
- [10] Gilles G, Hammami W, Libertiaux V, Cazacu O, Yoon JH, Kuwabara T et al. Experimental characterization and elasto-plastic modeling of the quasi-static mechanical response of TA-6V at room temperature. *Int. J. Solids Struct* 2011; **48**, 1277-89.
- [11] Cazacu O, Plunkett B, Barlat, F. Orthotropic yield criterion for hexagonal closed packed materials. *Int. J. Plasticity* 2006; **22**: 1171-94.
- [12] Hill R. The essential structure of constitutive laws for metal composites and polycrystals. *J Mech Phys Solids* 1967; **15**: 79–95.
- [13] Mandel J. *Plasticite classique et viscoplasticite*. CISM Courses and Lectures, Vol.97, International Center for Mechanical Sciences, Springer-Verlag, Wien-New York; 1972.
- [14] Tvergaard V. Influence of voids on shear band instabilities under plane strain conditions. *Int J Fracture* 1981; **17** : 389–407.
- [15] Aravas N. On the numerical integration of a class of pressure-dependent plasticity models. *Int J Numer Methods Eng* 1987; **24**: 1395–1416.
- [16] ABAQUS. *User's Manual for Version 6.8. Volumes I–V*. Dassault Systemes Simulia Corp., Providence, RI.; 2009.
- [17] Tirry, W, Nixon ME, Cazacu O, Coghe F, Rabet L. The importance of secondary and ternary twinning in compressed Ti. *Scripta Mater* 2011; **64**:840-3.
- [18] Revil-Baudard B, Cazacu O. The effect of the matrix tension-compression asymmetry on damage evolution in porous plastic solids. *J Mech Phys Solids* 2012 (submitted for publication).